PVP 12

Code: AE1T1, CE1T2, CS1T2, EC1T1, EE1T2, EM1T2, IT1T2, ME1T2

I B.Tech - I Semester – Regular Examinations February - 2014

ENGINEERING MATHEMATICS - I

(Common to all branches)

Duration: 3 hours — Marks: 5x14=70

Answer any FIVE questions. All questions carry equal marks

1. a) Solve $\frac{dy}{dx} + x\,sin2y = x^3 cos^2 y$ — 7 M

   b) Find the orthogonal trajectories of the family of confocal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1, \lambda$ being the parameter. — 7 M

2. a) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$ — 7 M

   b) Solve $(D^2 - 4)y = \cosh(2x - 1) + 3^x$ — 7 M

3. a) Find the Laplace Transform of

   (i) $te^{-2t} \sin 3t$ — 3 M

   (ii) $\frac{\sin t}{t}$ — 4 M

   b) Express $f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \cos 2t, & \pi < t < 2\pi \\ \cos 3t, & t > 2\pi \end{cases}$ in terms of unit step function and hence find the Laplace transform. 7 M

4. a) Find the inverse Laplace transform of

   (i) $\frac{s}{(s-3)(s^2+4)}$ 3 M

   (ii) $\log\left(\frac{s+a}{s+b}\right)$ 4 M

   b) Using the Laplace Transform Solve 7 M

   $(D^2 + 9)x = Cos\ 2t\ , given\ \ x(0) = 1, x(\pi/2) = -1.$

5. a) Using mean value theorem prove that

   $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$ if $0 < a < b < 1$. 7 M

   b) Find the dimensions of the rectangular box, open at the top of maximum capacity whose surface area is 432 sq.cm. 7 M

6. a) By using the transformation x + y = u, y = uv show that

   $\int_0^1 \int_0^{1-x} e^{\left(\frac{y}{x+y}\right)} dydx = \frac{1}{2}(e-1).$ 7 M

   b) Find the volume bounded by the XY-plane, the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 3$. 7 M

7. a) Show that $\nabla(\nabla \bullet (\frac{\bar{r}}{r})) = \frac{-2\bar{r}}{r^3}$ 4 M

   b) Prove that Curl (grad Φ)=0. 5 M

c) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at the point (1, -2, 1) in the direction of the vector $2\bar{i} - \bar{j} - 2\bar{k}$ 5 M

8. a) Evaluate $\int_S \bar{f} \bullet \bar{n}\, ds$ where $\bar{f} = 6z\bar{i} - 4\bar{j} + y\bar{k}$ and S is the portion of the plane 2x+3y+6z = 12 in the first octant. 6 M

b) Verify Stoke's theorem for $\bar{f} = (x^2 + y^2)\bar{i} - 2xy\bar{j}$ taken around the rectangle bounded by the lines $x = \pm a$ and y=0, y=b. 8 M